Applying Ideas from Homogeneity Analysis to Visualize Similarity Data

Nees Jan van Eck   Ludo Waltman

Econometric Institute, Erasmus School of Economics
Erasmus University Rotterdam
P.O. Box 1738, 3000 DR Rotterdam, The Netherlands
Email: {nvaneck,lwaltman}@few.eur.nl

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Outline of the presentation

Homogeneity analysis

VOS

Empirical comparison between VOS, MDS, and DAM

Application to a large data set

Conclusions

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- A method for visualizing the relations between categories and objects

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Objective

- To provide a low-dimensional visualization in which categories and objects are located in such a way that each category is the centroid of all objects that score on the category

Constrained optimization problem

Objective function

$$\sigma(\mathbf{X}, \mathbf{Y}; \mathbf{G}) = \sum_{i<j} g_{ij} \|\mathbf{x}_i - \mathbf{y}_j\|^2$$

where $\mathbf{x}_i$ and $\mathbf{y}_j$ denote, respectively, the location of object i and category j and $g_{ij} \in \{0, 1\}$ denotes whether or not object i scores on category j

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Constraints

$$\begin{aligned} \mathbf{X}'\mathbf{X} &= n\mathbf{I} \\ \mathbf{1}'\mathbf{X} &= 0 \end{aligned}$$

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- An abbreviation for *visualization of similarities*
- A new method for visualizing similarities between objects, based on ideas similar to homogeneity analysis

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Objective

- To provide a low-dimensional visualization in which objects are located in such a way that the distance between any pair of objects reflects their similarity as accurately as possible

Mathematical notation

Input

- n: number of objects
- m: number of dimensions of the solution
- $\mathbf{S} = (s_{ij})$: $n \times n$ similarity matrix (measured on a ratio scale)

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Mathematical notation

Input

- n: number of objects
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Output

- $\mathbf{X}$: $n \times m$ coordinate matrix
  $\mathbf{x}_i = (x_{i1}, \ldots, x_{im})$ contains the coordinates of object i

Constrained optimization problem

Objective function

$$E(\mathbf{X};\mathbf{S}) = \sum_{i<j} s_{ij} \|\mathbf{x}_i - \mathbf{x}_j\|^2$$

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- VOS:

$$\sum_{i<j} \|\mathbf{x}_i - \mathbf{x}_j\| = n(n-1)$$

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Motivation for the objective function

- Ideal coordinates of object i

$$c_i(\mathbf{X}, \mathbf{S}) = \frac{\sum_{j \neq i} s_{ij} \mathbf{x}_j}{\sum_{j \neq i} s_{ij}}$$

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$$c_i(\mathbf{X}, \mathbf{S}) = \frac{\sum_{j \neq i} s_{ij} \mathbf{x}_j}{\sum_{j \neq i} s_{ij}}$$

- Objective function for object i when the coordinates of all other objects are fixed

$$E_i(\mathbf{x}_i; \mathbf{X}, \mathbf{S}) = \sum_{j \neq i} s_{ij} \|\mathbf{x}_i - \mathbf{x}_j\|^2$$

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- VOS seems to have the tendency to locate objects close to their ideal coordinates

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Relationship between VOS and MDS

Objective function of weighted ratio MDS

$$\sigma(\mathbf{X}; \mathbf{D}, \mathbf{W}) = \sum_{i<j} w_{ij}(d_{ij} - \|\mathbf{x}_i - \mathbf{x}_j\|)^2$$

where d_{ij} denotes the dissimilarity between objects i and j and w_{ij} denotes the weight of objects i and j

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Let $s_{ij} > 0$ for all i and j $(i \neq j)$.
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VOS and weighted ratio MDS are then equivalent in the sense that solutions from these methods differ only by a multiplicative constant.

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Empirical comparison between VOS, MDS, and DAM (1)

Real-world bibliometric data sets

- Cocitation data of 376 journals in the field of economics and management (obtained from CWTS, Leiden University)
- Co-occurrence data on 332 concepts from the field of computational intelligence (obtained from Elsevier Scopus)
- In each data set around 75% of the similarities equal zero

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Methods

- VOS
- Unweighted ordinal MDS (SPSS PROXSCAL)
- Distance association model (DAM; De Rooij and Heijser, 2005)

Empirical comparison between VOS, MDS, and DAM (2)

VOS/MDS

- Co-occurrences/cocitations normalized using

$$s_{ij} = \frac{c_{ij}}{\left(\sum_k c_{kj}\right)\left(\sum_k c_{ik}\right)}$$

where c_{ij} denotes the co-occurrence/cocitation of objects i and j ($c_{ii} = 0$)

Empirical comparison between VOS, MDS, and DAM (2)

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DAM

- One-mode DAM with symmetric associations is used
- Row and column margins are assumed equal
- Distances are transformed into associations using the Gaussian function
- Likelihood under independent Poisson sampling is maximized

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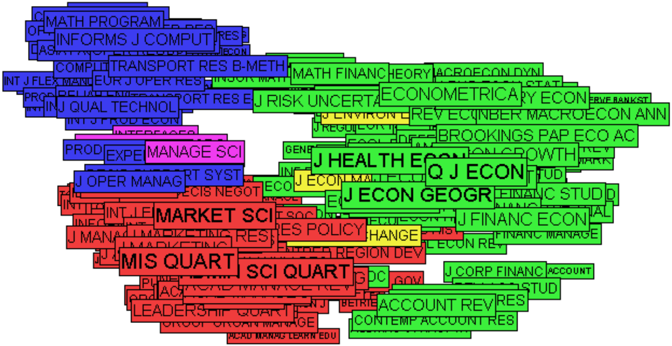
Journal cocitation map constructed using VOS

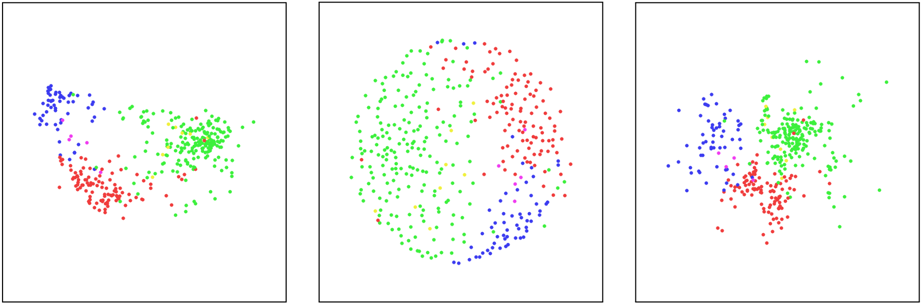


Journal cocitation map constructed using MDS

Journal cocitation map constructed using DAM

Comparison of journal cocitation maps

VOS | MDS | DAM



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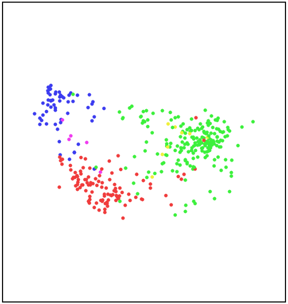
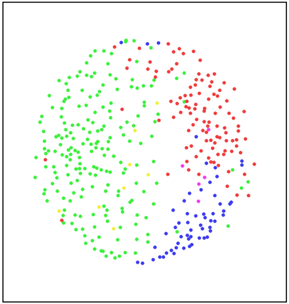
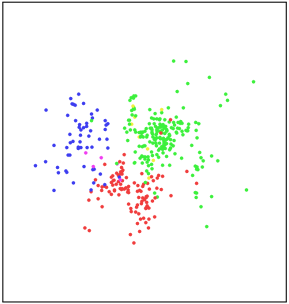
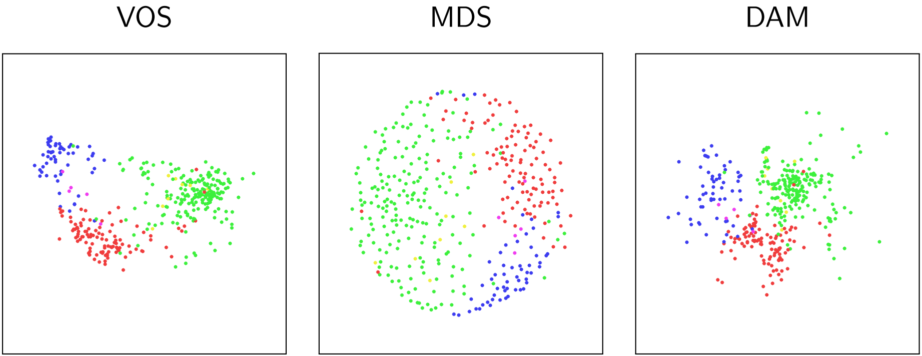






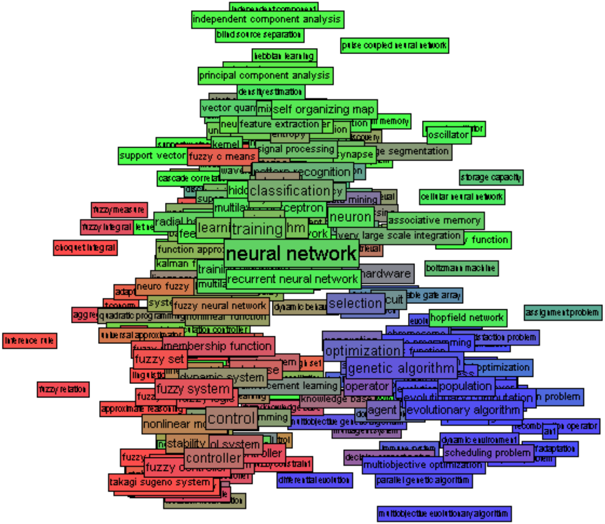
- MDS locates all objects roughly uniformly distributed within a circle

Comparison of journal cocitation maps



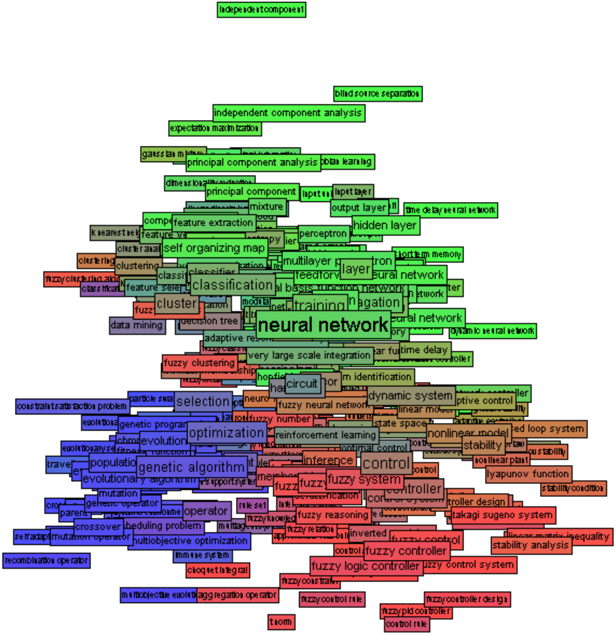
- MDS locates all objects roughly uniformly distributed within a circle
- VOS better separates the fields of economics, management, and operations research from each other than MDS and DAM

Concept co-occurrence map constructed using VOS



Concept co-occurrence map constructed using MDS

Concept co-occurrence map constructed using DAM



Comparison of concept co-occurrence maps

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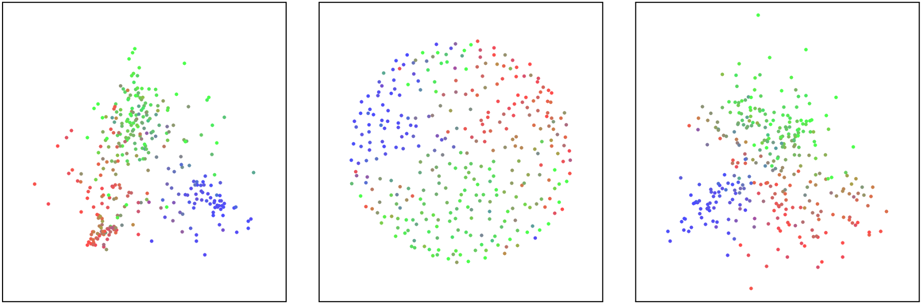

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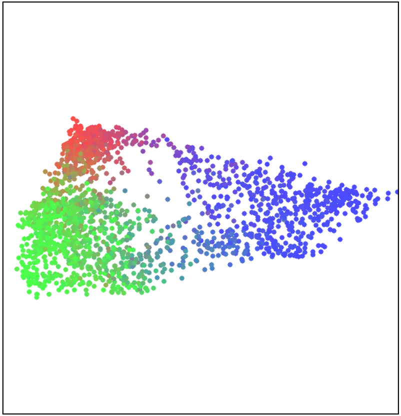
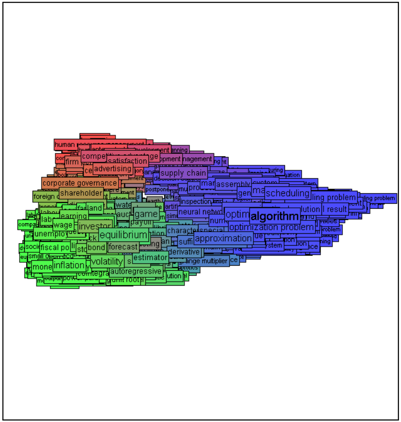
Application to a large data set

- Co-occurrence data on 2000 concepts from the field of economics and management (obtained from CWTS, Leiden University)
- Around 89% of the similarities equal zero





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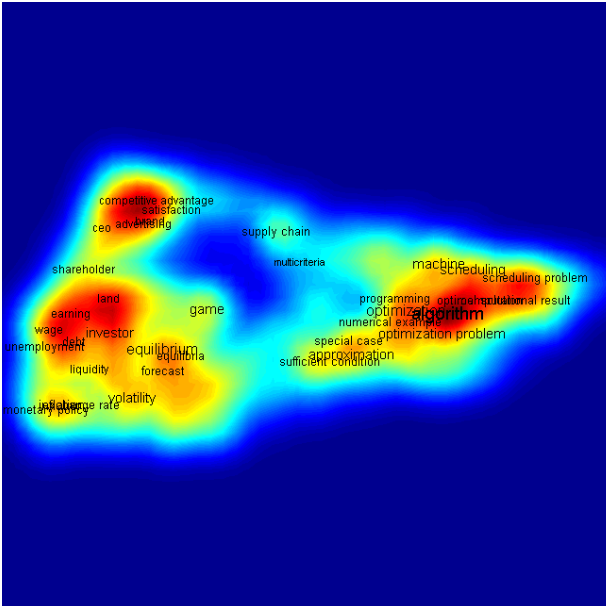
Concept density map



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Empirical comparison between VOS, MDS, and DAM

- Unweighted ordinal MDS locates objects roughly uniformly distributed within a circle
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Future research

- The effect on homogeneity analysis of using the VOS constraint

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More information on VOS

References to our papers and a computer implementation of VOS are available at:

http://www.neesjanvaneck.nl/vos/

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